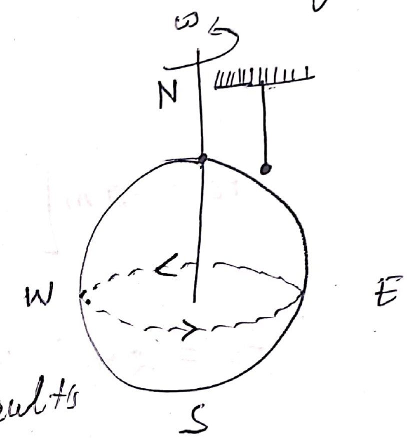
Foucault's Pendulum :->

A Foucault's Pendulum is similar to a simple pendulum in which a heavy bob is carried by a very long wire suspended from a rigid support is to have equal freedom of oscillation in any direction.

It proof that the earth is rotating from west to east and complete one cycle in 24 hours. It observed by Leon Foucault in 1851.

To proof this Foucault use 28 kg bob carried by 75 meter long steal wire.



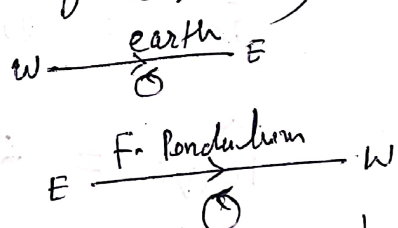
we know that time period of simple pendulum is

$$T_0 = \frac{2\pi}{\omega}$$

Now time period of ~~f~~ Foucault's Pendulum at north pole is

$$T = \frac{2\pi}{-\omega}$$

(–ve sign indicate the direction of pendulum opposite to the direction of earth)



$$T_{FP} = \frac{2\pi}{\omega \sin\phi} \quad \text{—— (1)}$$

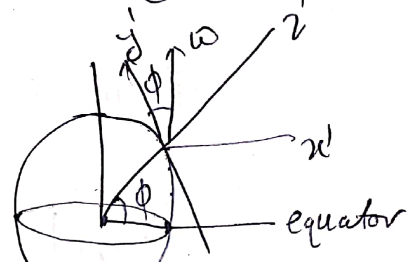
for earth rotation

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24}$$

$$\omega = \frac{2\pi}{24} \Rightarrow \frac{1}{\omega} = \frac{24}{2\pi}$$

$$T_{FP} = \frac{\cancel{2\pi}}{\sin\phi} \times \frac{24}{\cancel{2\pi}}$$

$$\boxed{T_{FP} = \frac{24}{\sin\phi}}$$



$$\omega = \omega \cos\phi\,\hat{j}' + \omega \sin\phi\,\hat{k}'$$

(No effect)

if $\phi = 0$ (at equator)

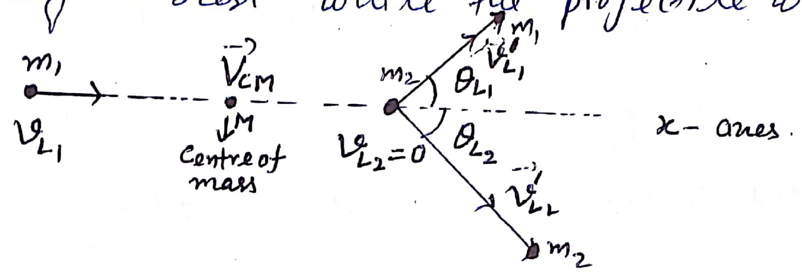
$T_{FP} = \infty$ (F.P not rotating)

if $\phi = 90°$ (Pole)

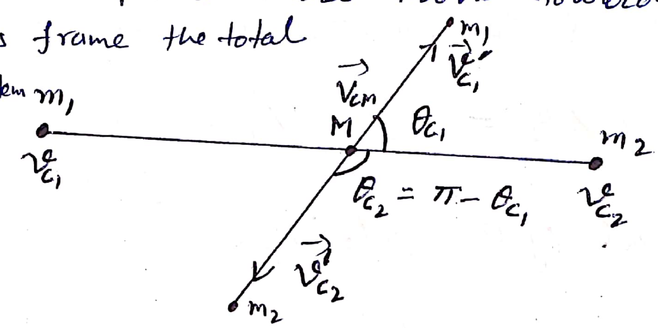
$$\boxed{T = 24\ H}$$

At pole angular velocity is same for earth and Foucault's Pendulum.

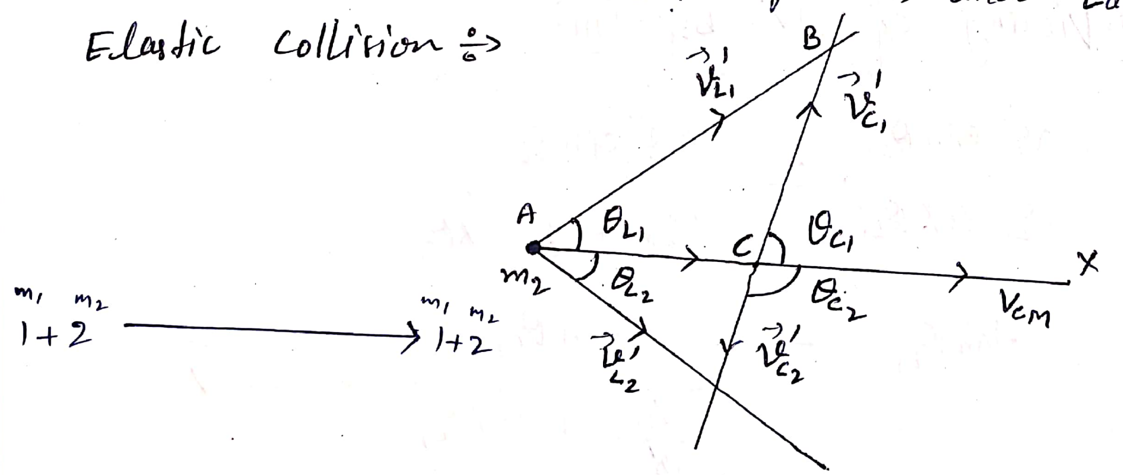
Laboratory frame or Lab frame :- The frame in which the target is initially at rest while the projectile are moving.



Centre of mass frame :- The frame in which the centre of mass of projectiles target system is at rest before and after Collision. Both the particles are move towards each other before collision, In this frame the total momentum of the system is zero,



Transformation between Centre of mass and Lab frame under Elastic collision :-



Let us consider a two particle system whose masses are m_1 and m_2 respectively.

v_{L_1} and v_{L_2} are the velocities of the particle 1 and 2 in the Lab frame before collision.

v'_{L_1} and v'_{L_2} are the velocities of the particle 1 and 2 in the Lab frame after collision.

v_{c_1} and v_{c_2} are the velocities of the particle 1 and 2 in the Centre of mass frame before collision.

v'_{c_1} and v'_{c_2} are the velocities of the particle 1 and 2 in the centre of mass frame after collision.

Now

$$\vec{v}_{L_1} = \vec{V}_{cm} + \vec{v}_{c_1} \quad \text{—— (I) before collision}$$

$$\vec{v}'_{L_1} = \vec{V}_{cm} + \vec{v}'_{c_1} \quad \text{—— (II) after collision.}$$

Taking the X-components of velocity

$$v'_{L_1} \cos\theta_{L_1} = V_{cm} + v'_{c_1} \cos\theta_{c_1} \quad \text{—— (III)}$$

Y-Component of the velocity.

$$v'_{L_1} \sin\theta_{L_1} = v'_{c_1} \sin\theta_{c_1} \quad \text{—— (IV)}$$

(V_{cm} only in x-directi
(Y component is zero)

Now dividing eq^n (IV) by (III)

$$\frac{\cancel{v'_{L_1}} \sin\theta_{L_1}}{\cancel{v'_{L_1}} \cos\theta_{L_1}} = \frac{v'_{c_1} \sin\theta_{c_1}}{V_{cm} + v'_{c_1} \cos\theta_{c_1}}$$

$$\tan\theta_{L_1} = \frac{\sin\theta_{c_1}}{\frac{V_{cm}}{v'_{c_1}} + \cos\theta_{c_1}} \quad \text{—— (V)}$$

By the definition of Centre of mass

$$\vec{V}_{cm} = \frac{m_1 \vec{v}_{L_1} + m_2 \vec{v}_{L_2}}{m_1 + m_2}$$

but $\vec{v}_{L_2} = 0$

$$\vec{V}_{cm} = \left(\frac{m_1}{m_1+m_2}\right)\vec{v}_{L_1}$$

$$\vec{v}_{L_1} = \left(\frac{m_1}{m_1+m_2}\right)\vec{v}_{L_1} + \vec{v}_{c_1}$$

$$\vec{v}_{c_1} = \left(\frac{m_2}{m_1+m_2}\right)\vec{v}_{L_1}$$

$$\frac{\vec{V}_{cm}}{\vec{v}_{c_1}} = \frac{m_1}{m_2}$$

$$\tan\theta_{L_1} = \frac{\sin\theta_{c_1}}{\frac{m_1}{m_2} + \cos\theta_{c_1}}$$

$$\boxed{\tan\theta_{L_1} = \frac{m_2\sin\theta_{c_1}}{m_1+m_2\cos\theta_{c_1}}}$$

In centre of mass frame the total momentum is zero.

$$\vec{P}_{c_1} + \vec{P}_{c_2} = 0$$

$$m_1\vec{v}_{c_1} + m_2\vec{v}_{c_2} = 0 \quad \text{—— (6)}$$

$$\vec{P}'_{c_1} + \vec{P}'_{c_2} = 0$$

$$m_1\vec{v}'_{c_1} + m_2\vec{v}'_{c_2} = 0 \quad \text{—— (7)}$$

$$m_1\vec{v}_{c_1} = -m_2\vec{v}_{c_2} \Rightarrow \vec{v}_{c_2} = -\frac{m_1}{m_2}\vec{v}_{c_1}$$

similarly $\vec{v}'_{c_2} = -\frac{m_1}{m_2}\vec{v}'_{c_1}$

This is elastic collision

Therefore K.E is conserved.

$$\frac{1}{2}m_1 v_{c_1}^2 + \frac{1}{2}m_2 v_{c_2}^2 = \frac{1}{2}m_1 v_{c_1}'^2 + \frac{1}{2}m_2 v_{c_2}'^2$$

$$m_1 v_{c_1}^2 + m_2 \times \left(\frac{m_1}{m_2}v_{c_1}\right)^2 = m_1 v_{c_1}'^2 + m_2\left(-\frac{m_1}{m_2}v_{c_1}'\right)^2$$

$$\cancel{\left(m_1 + \frac{m_1^2}{m_2}\right)} v_{c_1}^2 = \cancel{\left(m_1 + \frac{m_1^2}{m_2}\right)} v_{c_1}'^2$$

$$v_{c_1}^2 = v_{c_1}'^2$$

$$|v_{c_1}| = |v'_{c_1}|$$

Similarly $\quad |v_{c_2}| = |v'_{c_2}|$

In elastic collision the magnitude of velocity in the centre of mass frame before and after the collision remain same.

$$\vec{V}_{cm} = \left(\frac{m_1}{m_1+m_2}\right)\vec{v}_{L_1}$$

$$\vec{v}_{c_1} = \left(\frac{m_2}{m_1+m_2}\right)\vec{v}_{L_1}$$

We define the relative velocity $\vec{v}$

$$\vec{v} = \vec{v}_{L_1} - \vec{v}_{L_2} \quad \text{——— (A)}$$

$$\vec{v} = \cancel{\vec{V}_{cm}} + \vec{v}_{c_1} - \cancel{\vec{V}_{cm}} - \vec{v}_{c_2}$$

$$\vec{v} = \vec{v}_{c_1} - \vec{v}_{c_2} \quad \text{——— (B)}$$

Centre of mass system momentum is zero.

$$\vec{P}_{c_1} + \vec{P}_{c_2} = 0$$

$$m_1\vec{v}_{c_1} + m_2\vec{v}_{c_2} = 0 \Rightarrow \vec{v}_{c_2} = -\frac{m_1}{m_2}\vec{v}_{c_1}$$

$$m_1\vec{v}'_{c_1} + m_2\vec{v}'_{c_2} = 0 \Rightarrow \vec{v}'_{c_2} = -\frac{m_1}{m_2}\vec{v}'_{c_1}$$

$$m_1\vec{v}_{c_1} + \cancel{m_2\vec{v}_{c_2}} = 0$$

$$m_2 v_{c_1} - \cancel{m_2 v_{c_2}} = m_2\vec{v} \qquad \times (m_2)$$

$$(+)$$

$$(m_1+m_2)\vec{v}_{c_1} = m_2\vec{v}$$

$$\vec{v}_{c_1} = \left(\frac{m_2}{m_1+m_2}\right)\vec{v}$$

$$|\vec{v}_{c_1}| = |\vec{v}'_{c_1}| = \left(\frac{m_2}{m_1+m_2}\right)|\vec{v}| \quad \text{——— (C)}$$

$$m_1\vec{v}_{c_1} + m_2\vec{v}_{c_2} = 0$$

$$\left(\frac{m_1 \cancel{m_2}}{m_1+m_2}\right)\vec{v}' = -\cancel{m_2}\vec{v}_{c_2}$$

$$\vec{v}_{c_2} = -\left(\frac{m_1}{m_1+m_2}\right)\vec{v}$$

$$\boxed{|\vec{v}_{c_2}| = |\vec{v}\,'_{c_2}| = \left(\frac{m_1}{m_1+m_2}\right) v} \quad \text{--- (9)}$$

$$\vec{v}\,'_{L_1} = \vec{V}_{cm} + \vec{v}\,'_{c_1}$$

$$\vec{v}\,'_{L_1} = \frac{m_1\vec{v}_{L_1} + m_2\vec{v}_{L_2}}{m_1+m_2} + \left(\frac{m_2}{m_1+m_2}\right)\vec{v}$$

$$\vec{v}\,'_{L_1} = \frac{\vec{P}_{L_1} + \vec{P}_{L_2}}{(m_1+m_2)} + \left(\frac{m_2}{m_2+m_2}\right)\vec{v}$$

$$m_1\vec{v}\,'_{L_1} = \left(\frac{m_1}{m_1+m_2}\right)(\vec{P}_{L_1} + \vec{P}_{L_2}) + \left(\frac{m_1 m_2}{m_1+m_2}\right)\vec{v}'$$

reduce mass $\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$

$$\frac{1}{\mu} = \frac{m_1+m_2}{m_1 m_2}$$

$$\mu = \frac{m_1 m_2}{m_1+m_2}$$

$$m_1\vec{v}\,'_{L_1} = \frac{m_1}{m_1+m_2}(\vec{P}_{L_1} + \vec{P}_{L_2}) + \mu\vec{v}$$

$$P'_{L_1} = \frac{m_1}{m_1+m_2}(\vec{P}_{L_1} + \vec{P}_{L_2}) + \mu\vec{v} \quad \text{--- (10)}$$

$$v'_{L_2} = \vec{V}_{cm} + \vec{v}'_{c_2}$$

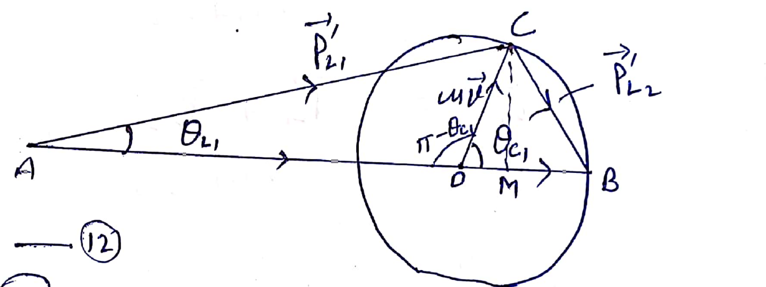
$$\vec{v}'_{L_2} = \frac{m_1 \vec{v}_{L_1} + m_2 \vec{v}_{L_2}}{m_1+m_2} - \frac{m_1}{m_1+m_2}\vec{v}$$

$$m_2 \vec{v}'_{L_2} = \frac{m_2}{m_1+m_2}\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right) - \left(\frac{m_1 m_2}{m_1+m_2}\right)\vec{v}$$

$$\vec{P}'_{L_2} = \frac{m_2}{m_1+m_2}\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right) - \mu\vec{v} \quad \text{——} (11)$$

From equation (10) and (11)

$$\boxed{\vec{P}'_{L_1} + \vec{P}'_{L_2} = \vec{P}_{L_1} + \vec{P}_{L_2}}$$



From equation (10) and (11)

$$\vec{AO} + \mu\vec{v} = \vec{P}'_{L_1} \quad \text{——} (12)$$

Compare with equation (10)

$$\vec{AO} = \frac{m_1}{m_1+m_2}\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right)$$

θ_{L_1} is the scattering angle in Lab frame.

θ_{c_1} is the scattering angle in the Centre of mass frame.

$$\vec{OC} + \vec{CB} = \vec{OB}$$

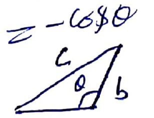
$$\vec{CB} = \vec{OB} - OC$$

$$\vec{P}'_{L_2} = \vec{OB} - \mu\vec{v} \quad \text{——} (13)$$

Compare with equation (11)

$$\vec{OB} = \frac{m_2}{m_1+m_2}\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right)$$

$\cos(180-\theta) = -\cos\theta$



$\cos\theta = \frac{a^2+b^2-c^2}{2ab}$

$$AC^2 = AO^2 + OC^2 - 2\,\overline{AO}\;\overline{OC}\cos(\pi - \theta_{c_1})$$

$$P'^2_{L_1} = \left(\frac{m_1}{m_1+m_2}\right)^2\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right)^2 + \mu^2 v^2 + 2\left(\frac{m_1}{m_1+m_2}\right)\left(\vec{P}_{L_1} + \vec{P}_{L_2}\right)\mu\vec{v}\cos\theta_{c_1}$$

$$\vec{P}_{L_2} = 0 \quad \because \vec{v}_2 = 0$$

$$\vec{v}' = \vec{v}_1 - \vec{v}_2 \Rightarrow \vec{v} = \vec{v}_1$$

$$\vec{v}_1 = \vec{v}$$

$$P'^2_{L_1} = \left(\frac{m_1}{m_1+m_2}\right)^2 P^2_{L_1} + \mu^2 v^2 + 2\left(\frac{m_1}{m_1+m_2}\right) \vec{P}_{L_1} \mu \vec{v} \cos\theta_{c_1}$$

$$P'^2_{L_1} = \left(\frac{m_1^2 v}{m_1+m_2}\right)^2 + \mu^2 v^2 + 2\left(\frac{m_1^2 v}{m_1+m_2}\right) \mu v \cos\theta_{c_1}$$

$$P'^2_{L_1} = \frac{m_1^2}{m_2^2}\left(\frac{m_1 m_2}{m_1+m_2} v\right)^2 + \mu^2 v^2 + 2\frac{m_1}{m_2}\left(\frac{m_1 m_2}{m_1+m_2}\right) v \mu v \cos\theta_{c_1}$$

$$m_1 v'^2_{L_1} = \left(\frac{m_1}{m_2}\right)^2 \mu^2 v^2 + \mu^2 v^2 + 2\frac{m_1}{m_2} \mu^2 v^2 \cos\theta_{c_1}$$

$$m_1^2 v'^2_{L_1} = \frac{m_1^2}{m_2^2} \times \left(\frac{m_1 m_2}{m_1+m_2}\right)^2 v^2 + \left(\frac{m_1 m_2}{m_1+m_2}\right)^2 v^2 + \frac{2m_1}{m_2}\left(\frac{m_1 m_2}{m_1+m_2}\right)^2 v^2 \cos\theta_{c_1}$$

$$\cancel{m_1^2} v'^2_{L_1} = \frac{\cancel{m_1^2} v^2}{(m_1+m_2)^2}\left[\frac{\cancel{m_1^2}}{\cancel{m_2^2}} \times m_1^2 \times \cancel{m_2^2} + \frac{\cancel{m_1^2} m_2^2}{\cancel{m_1^2}} + \frac{2}{\cancel{m_2}\,\cancel{m_1}} \times \cancel{m_1^2}\,\cancel{m_2^2} \cos\theta_{c_1}\right]$$

$$v'^2_{L_1} = \frac{v^2}{(m_1+m_2)^2}\left(m_1^2 + m_2^2 + 2m_1 m_2 \cos\theta_{c_1}\right)$$

$$\boxed{v'_{L_1} = \frac{v}{(m_1+m_2)}\left(m_1^2 + m_2^2 + 2m_1 m_2 \cos\theta_{c_1}\right)^{1/2}}$$

Now

$$CB^2 = OC^2 + OB^2 - 2\,\overline{OC}\,\overline{OB}\cos\theta_{c_1}$$

$$P'^2_{L_2} = \mu^2 v^2 + \left(\frac{m_1 m_2}{m_1+m_2} v\right)^2 - 2\mu v \frac{m_1 m_2 v}{m_1+m_2} \cos\theta_{c_1}$$

$$P'^2_{L_2} = \mu^2 v^2 + \mu^2 v^2 - 2\mu^2 v^2 \cos\theta_{c_1}$$

$$P'^2_{L_2} = \mu^2 v^2 (1 + 1 - 2\cos\theta_{c_1})$$

$$\cancel{m_2^2} v'^2_{L_2} = \frac{m_1^2 \cancel{m_2^2}}{(m_1+m_2)^2} v^2 (2 - 2\cos\theta_{c_1})$$

$$v'^2_{L_2} = \frac{2 m_1^2 v^2}{(m_1+m_2)^2} (1 - \cos\theta_{c_1})$$

$$v'^2_{L_2} = \frac{2 m_1^2 v^2}{(m_1+m_2)^2} (\cancel{1} - \cancel{1} + 2\sin^2 \theta_{c_1}/2)$$

$$v'^2_{L_2} = \frac{4 m_1^2 v^2}{(m_1+m_2)^2} \sin^2 \frac{\theta_{c_1}}{2}$$

$$\boxed{v'_{L_2} = \frac{2 m_1 v}{(m_1+m_2)} \sin \frac{\theta_{c_1}}{2}}$$